



## A Stochastic Model of an Infectious Disease, Based on the Birth-and-Death-with-Immigration Process

**DR. HISASHI KOBAYASHI**

*THE SHERMAN FAIRCHILD UNIVERSITY PROFESSOR, EMERITUS  
PRINCETON UNIVERSITY*

### Abstract

Why are the epidemic patterns of COVID-19 so different among different cities or countries which are similar in their populations, medical infrastructures, and people's behavior? Why are forecasts or predictions made by so-called experts often grossly wrong, concerning the numbers of people who get infected or die?

The purpose of this study is to better understand the stochastic nature of an epidemic disease and answer the above questions. Much of the work on infectious diseases has been based on "SIR deterministic models" (Kermack and McKendrick: 1927). We will explore a new stochastic model that can capture the essence of the seemingly erratic behavior of an infectious disease.

The stochastic model we propose is based on the "birth-and-death process with immigration" (BDI for short), which was originally proposed in the study of population growth or extinction of some biological species. The BDI process model, however, does not seem to have been investigated by the epidemiology community. The BDI process is one of a few birth-and-death processes, which we can solve analytically. We revisit the partial differential equation (PDE) for the probability generating function (PGF) of the time nonhomogeneous BDI process and derive a closed form solution

We clarify the relationships among the basic reproduction number, effective reproduction number, the exponential growth/decay parameter, the proportion of vaccinated population, vaccine effectiveness, and the behavioral factor of the public.

We also derive maximum-likelihood estimation of the model parameters, and present surprisingly simple formulas for estimating our model parameters, and the exponential growth/decay parameter. We identify some challenges in parameter estimation arising from two important characteristics of COVID-19 and its variants. First, a significant proportion of infections are asymptomatic. Second, each infection has its incubation period, which is a random variable.

### Brief Bio-Sketch

Hisashi Kobayashi was born in Tokyo, Japan in 1938, and received his B.E. (1961) and M.E. (1963) in electrical engineering from the University of Tokyo. He then worked for Toshiba Corp in its radar system department (1963-65). He came to Princeton University as an Orson Desaix Munn Fellow in 1965 and received his Ph.D. in 1967.

From 1967 to 1986 he worked for the IBM Research Division. In 1969-70 he invented a high-density digital recording technology, widely known as PRML (partial-response, maximum-likelihood), which has become a de-facto industry standard for virtually all digital recording products, both magnetic and optical. He also worked on analytic modeling techniques for computer performance evaluation. He is known for the computationally efficient algorithms he devised for Markovian queuing networks, and for the introduction of diffusion process approximation in general queueing networks. He was the founding Director of IBM Japan Science Institute (1982-86)

In 1986, he joined Princeton University as the Sherman Fairchild University Professor and as Dean of the School of Engineering and Applied Science (1986-91). Among his research accomplishments at Princeton are the loss network theory and computationally efficient algorithms (with B. L. Mark); iterative decoding for general concatenated systems (with J. Bajcsy); and wireless geolocation algorithms (with Y. Qi).

He authored three books: Modeling and Analysis (1978, Addison Wesley); System Modeling and Analysis (with B. L. Mark; 2008, Pearson/Prentice Hall), and Probability, Random Processes and Statistical Analysis (with B. L. Mark and W. Turin; 2012, Cambridge University Press).

Among the awards and honors he has received are: IBM Outstanding Contribution Award (1974), Fellow of IEEE (1977), Senior U.S. Scientist Award from the Alexander von Humboldt Foundation, West Germany (1979), Silver Core Award from IFIP (1980), Engineering Academy of Japan (1992); Technology Award from Eduardo Rhein Foundation, Germany (2005); C&C Prize from NEC C&C Foundation, Japan (2012); and Honorary Doctorate Degree from Ghent University, Belgium (2018).

**MAR. 4, 2:00 PM ON ZOOM**

**Zoom Meeting Link:**

**<https://gmu.zoom.us/j/95160126923>**