Mitigation of NLOS Effects in TOA Positioning

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Abstract — **It is well known that in an environment where the LOS (Line-of-sight) signal is extremely weak or non-existent the accuracy of geolocation will be considerably degraded, which is often the case in a cellular system. So mitigation of the NLOS effects is an important issue in wireless geolocation. Several methods have been discussed in the literature to address this issue. However, these methods may not provide reliable performance at practical computation cost in typical situations. We propose a new and promising approach to address this problem. We formulate it as a constrained optimization problem, which can be solved by the so-called sequential simplex method (SSM). Simulation results show that the accuracy of less than $100m$ can be obtained in most cases with reasonable computation load.**

I. Introduction

Geolocation with multiple base stations' TOAs (Time-of-arrival) in an LOS (Line-of-sight) environment can be solved by the least square (LS) method [1] with an acceptable accuracy. Yet the performance will be considerably degraded when NLOS (Non-line-of-sight) TOAs exist, which is often the case in a cellular system. So mitigation of NLOS effects is an important issue in wireless geolocation. Several methods [2, 1] have been proposed to mitigate NLOS effects. However, the existing methods may not be able to provide reliable performance at practical computation cost in typical situations.

Our task is therefore to mitigate the NLOS effects with reliable performance and reasonable computation load. Still based on the LS approach, our proposed scheme is to minimize an objective function with two types of constraint. One set of constraints (see Eq.(3)) is to address the NLOS effect that the range measurement with NLOS error is always larger than the true range. The other constraint (see Eq.(4)) is to take into account that the MS will be either stationary or moving with speed bounded by some finite speed, thus the incremental change of the MS position from its position at the previous measurement instant is bounded by some finite number. For the sake of mathematical clarity, we define a modified objective function (see Eq.(6)). The estimation of the MS position in the NLOS environment is therefore treated as an unconstrained minimization problem. We have found the sequential simplex method (SSM) is a simple but powerful tool for our problem. It is outlined in detail in the context of two dimensional minimization problem in [4]. As seen in our discussion and simulation examples, it is guaranteed that an optimal estimate of the MS position at a given moment t falls always within the feasible region defined by the two constraints with positioning error less than $100m$ in typical situations.

The rest of the paper is organized as follows. In Section II, we formulate the problem as a constrained optimization problem, which can be solved by SSM, discussed briefly in Section III. Simulation results are presented in Section IV. In Section V, we make a conclusion and relate some further work.

II. Problem Formulation

Let $(x(t), y(t))$ be the MS position at time t and the B BSs are located at $\{(x_b, y_b)\}_{b=1}^{B}$. Let $\{r_b(t)\}_{b=1}^{B}$ be the noisy range measurements obtained from the BS' TOA measurements at time t. The noise comes from two sources. One is formulated as Gaussian variables $\{n_b(t)\}_{b=1}^{B}$ with distribution $N(0, \sigma^2)$, and the other represents an error in the TOA estimate introduced by the NLOS propagation, and is always positive, which we denote as $\{L_b(t)\}_{b=1}^{B}$. So the noisy range measurement at base station b can be expressed as

$$r_b(t) = \sqrt{(x_b - x(t))^2 + (y_b - y(t))^2} + n_b(t) + L_b(t), \quad \text{for } b = 1, 2, \cdots, B. \tag{1}$$

We will drop parameter t when it causes no confusion. The following objective function is then defined

$$F(x, y) = \sum_{b=1}^{B} w_b \cdot \left(\sqrt{(x_b - x)^2 + (y_b - y)^2} - r_b\right)^2, \tag{2}$$

where w_b is a weight that reflects the reliability of a measurement at base station b. Since $L_b(t) \geq 0$ and $n_b(t) \leq 3\sigma$ at the confidence level of more than 95% for each b, by some straightforward manipulation of Eq.(1), we get the first set of constraints

$$C_b(x, y) = r_b + 3\sigma - \sqrt{(x_b - x)^2 + (y_b - y)^2} \geq 0, \quad \text{for } b = 1, 2, \cdots, B. \tag{3}$$

These constraint simply express the fact that the range measurement error caused by the NLOS propagation is always positive. The other type of constraint is formulated by considering that the MS will be either stationary or moving with speed bounded by some finite speed, so the incremental change in the MS position from its position at the previous measurement instant is also bounded by some finite number, denoted as $\Delta(t)$ here. This constraint can be defined as

[1] This work has been supported, in part, by the New Jersey Center for Wireless Telecommunications (NJCWT), NTTDoCoMo Co. and Digital Angel.Net, Inc.

$$C_{B+1}(x,y) = \Delta(t) - [(x(t-\delta t) - x(t))^2 + (y(t-\delta t) - y(t))^2]^{1/2} \geq 0 \quad (4)$$

Denote the maximum speed of MS during $[t-\delta t, t]$ as $v_m(t)$ and the maximum positioning error of MS at time t as $\phi(t)$. The explicit expression of $\Delta(t)$ is

$$\Delta(t) = v_m(t) \cdot \delta t + \phi(t - \delta t). \quad (5)$$

Our goal is to minimize the objective function Eq.(2) subject to two types of constraints, i.e., Eqs.(3) and (4). This is a constrained optimization problem that can theoretically be solved by any standard gradient techniques [1]. We found, however, they are not guaranteed to converge to a position with acceptable error from the true MS position in an NLOS propagation environment. We then found in [4] a simple but powerful iterative method, known as *sequential simplex method* (SSM). As seen in the simulation examples, the final converging point (i.e., an optimal estimate of the MS position at a given moment t) falls always within the feasible region defined by the $B+1$ constraint equations (3) and (4) at time t, providing an acceptable positioning accuracy.

For the sake of mathematical clarity, the original problem is recast into the problem of minimizing the following unconstrained objective function

$$\tilde{F}(x,y) = \sum_{b=1}^{B} w_b \cdot (\sqrt{(x_b - x)^2 + (y_b - y)^2} - r_b)^2 + G(C_1(x,y), \cdots, C_B(x,y), C_{B+1}(x,y)), \quad (6)$$

where

$$G(C_1, \cdots, C_B, C_{B+1}) = \begin{cases} 0 & \text{if all } C_i \geq 0, \\ & \text{for } i = 1, \cdots, B, B+1. \\ +\infty & \text{otherwise} \end{cases}$$

In order to complete our discussion on the SSM technique, we now address the question of how to select the initial three points for this iterative method.

Since the SSM is an iterative method, we must provide an initial set of points. In our problem, the initial points must satisfy the constraints (3) and (4). We can obtain appropriate points by minimizing the following function

$$H(x,y) = \sum_{i=1}^{B+1} I(C_i(x,y)) \cdot C_i(x,y), \quad (7)$$

where

$$I(C_i) = \begin{cases} 0 & \text{if } C_i \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

The initial points for this minimization problem can be, in theory, arbitrarily chosen. To simplify computations, a better choice will be to take the MS position estimate obtained at the previous measurement instant $t - \delta t$ as the initial point at time t, which is what exactly we have done in our simulations.

III. Sequential Simplex Method

The SSM is an iterative method for optimization. It involves an iterative evaluation of the given objective function itself in contrast to evaluation of derivatives of the objective function performed in a typical gradient method. The SSM thus has an advantage that it can accommodate discontinuity in the objective function. This is the case in our problem: the modified objective function Eq. (6) would become discontinuous at boundary points (x, y) when either of the constraint Eqs. (3) and (4) is violated. The detailed description of this method can be found in [4].

IV. Simulation Results

We now discuss some simulation examples to study the performance of our proposed geolocation method in an NLOS propagation environment.

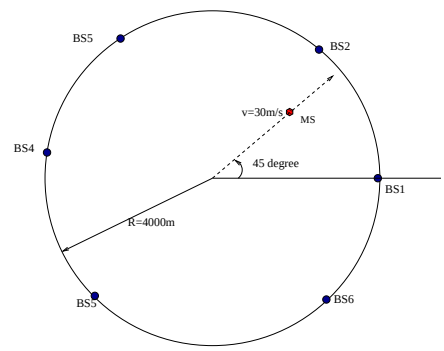


Figure 1: Configuration of BSs and MS in the simulation example.

Assume that there are six base stations located along a circle with radius of $4000m$. The angular separation between adjacent base stations is $\pi/3$. A mobile station is assumed to be located somewhere within the circle, moving with the speed $30m/s$ along a straight line at 45 degree from the horizontal-axis. The configuration is shown in Fig. 1. A CDMA system is assumed. The standard deviation σ of Gaussian error (see Eq.(1)) is related to chip duration T_c according to

$$6 \cdot \sigma = c \cdot T_c$$

where $c = 3 \times 10^8 m/s$ is the speed of light.

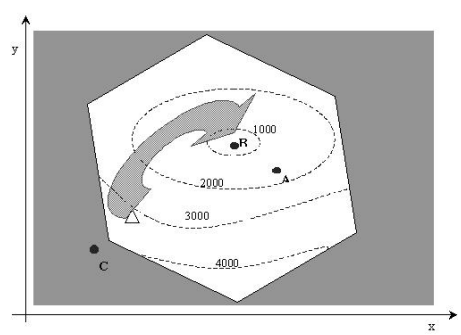
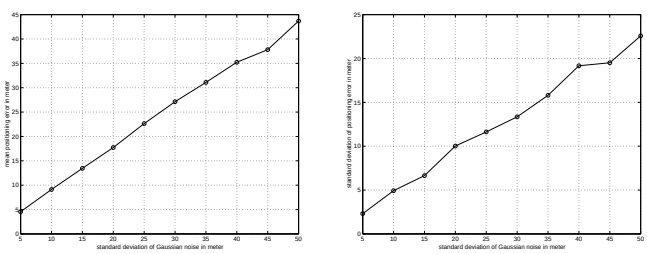


Figure 2: Illustrative picture of our proposed geolocation method.

Fig. 2 illustrates how the proposed method works. We have a set of noisy range measurements $\{r_b\}_{b=1}^{B}$ at time t, as defined

in Eq.(1), and the MS position estimate at the previous measurement instant $t-\delta t$, which is point C. From the constraints of Eqs.(3) and (4), we can obtain the feasible region for the MS position at time t, which is the unshaded region. The initial set of points, which is close to the unshaded region boundary, is obtained by minimizing the objective function defined by Eq.(7) with the sequential simplex method. The big arrow represents intermediate searching steps in minimization of the objective function given by Eq.(6). The labels on the contours correspond to values of the objective function. The point B is the MS position estimate at time t after convergence of the iterative steps. The true MS position is shown as point A. The difference between the true MS position A and its estimate B is the error associated with this least-square based method. The difference can be made within an acceptable limit by choosing some appropriate parameter in the sequential simplex method.

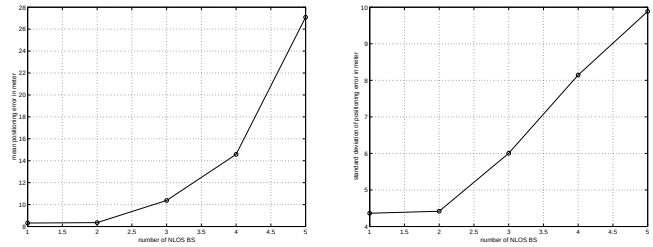
Some of our simulation results are shown in Fig. 3 and Fig. 4. The MS position estimate at the previous measurement $t-\delta t$ is $2000(\cos\pi/4, \sin\pi/4) = (1414m, 1414m)$. The true MS position at time t is $2025(\cos\pi/4, \sin\pi/4) = (1449.8m, 1449.8m)$. The extra range introduced by NLOS propagation is assumed to be larger than $50m$. The maximum positioning error at time $t-\delta t$ is set to be $30m$. Each point in the plots is produced by averaging 200 simulation runs. In Fig. 3, we assume that the range measurements at two base stations, BS2 and BS4, are corrupted by NLOS propagation. Fig. 3(a) and Fig. 3(b) are the mean and the standard deviation of the positioning error, respectively, in meter versus the standard deviation of Gaussian error in meter. It is shown that both the mean and the standard deviation of positioning error increases as the level of Gaussian error becomes higher, which is also intuitively reasonable. In Fig. 4, the standard deviation of Gaussian error is fixed at $10m$. The x-axis now is the number of base stations with NLOS range measurement, while the y-axis is same as in Fig. 3. The performance is degraded while the number of NLOS base stations increases.



(a) (b)

Figure 3: Performance of our proposed geolocation method in an NLOS environment with different Gaussian error levels. There are six base stations, two of which provide NLOS measurements.

As we may notice, the position error is less than $100m$ in all cases, even in some harsh situations, e.g., only one BS with LOS measurement. The good performance comes from the fact that constraints (3) and (4) limit the position estimate in a tight feasible region, as illustrated in Fig. 2. However, if some unfavorable situation, e.g., five NLOS BSs in our case, persists for several measurement instants, the performance is expected to severely degrade. The reason is that the maximum positioning error $\Delta(t)$ (see Eq. (5)) will accumulate to a large number during an adaptive procedure, which leads to loose boundaries from the constraint. This seems an unavoidable issue in many position tracking problems. We will make quantitative analysis in our future study.



(a) (b)

Figure 4: Performance of our proposed geolocation method in an NLOS environment with different numbers of NLOS BS's. There are six base stations. The standard deviation of Gaussian noise is $10m$.

V. Conclusion and Further Work

In this paper, we propose an approach to mitigate the NLOS effect in TOA positioning, which is formulated as the problem of minimizing an objective function under two types of constraints. The sequential simplex method is applied as the optimization technique. Simulation results show that the accuracy of $100m$ can be achieved in typical cases. Our further work includes a simulation study of more realistic situations and an investigation of how to update some parameters in adaptive geolocation tracking procedure.

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